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Transport of charge-density waves in the presence of disorder: Classical pinning vs quantum localization

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We consider the interplay of the elastic pinning and the Anderson localization in the transport properties of a charge-density wave in one dimension, within the framework of the Luttinger model in the limit of strong repulsion. We address a conceptually important issue of which of the two disorder-induced phenomena limits the mobility more effectively. We argue that the interplay of the classical and quantum effects in transport of a very rigid charge-density wave is quite nontrivial: the quantum localization sets in at a temperature much smaller than the pinning temperature, whereas the quantum localization length is much smaller than the pinning length.

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A confluence of ideas formulated for mesoscopic disordered electron systems on one side and for strongly correlated clean systems on the other has brought forth a new field—mesoscopics of strongly correlated electron systems. Prominent examples of such systems include charge-density waves (CDWs), Wigner crystals, and Luttinger liquids. Low-energy excitations in these systems are of essentially collective nature and are described in terms of elastic waves. In the presence of static disorder, the conductivity of the systems is strongly suppressed at low temperatures, which is commonly referred to as pinning, or localization.

A key concept in the mesoscopics of disordered electron systems is that of the Anderson localization [1]. This phenomenon is due to the quantum interference of multiply-scattered electron waves on spatial scales larger than the localization length $\xi_{\rm loc}$. The localization is destroyed by inelastic electron-electron (e-e) scattering on the scale of the dephasing length. The notions of weak localization and dephasing due to e-e scattering, established for Fermi-liquid systems [2], have recently been shown to be also applicable to Luttinger liquids [3].

On the other hand, considering the interplay of disorder and interaction in the opposite limit of a very strong Coulomb interaction, one arrives at the concept of *pinning* of elastic waves on the spatial scale of the pinning length $\xi_{\rm pin}$ [4]. This concept has a long history [5, 6] and applies not only to CDWs and similar electron systems but, more generally, to every elastic object in a random environment, ranging from domain walls in ferromagnets and ferroelectrics to vortex lattices in type-II superconductors. However, as far as strongly correlated electron systems are concerned, the notions of Anderson localization and pinning are often viewed in the literature as essentially synonymous. Specifically, the Anderson localization of electrons and the pinning of CDWs are thought of as two sides of the same phenomenon which gradually evolves with changing strength of e-e interaction.

However, it is useful to recall that the physics of localization and pinning, as we know them from the pioneering works by Anderson [1] and Larkin [4], respectively, is distinctly different: the localization is a quantum phenomenon, whereas the pinning is essentially classical. This distinction has important consequences. In particular, in classical elastic systems at vanishing coupling to the external thermal bath, any inelastic scattering results in the activated temperature behavior of the mobility at low T. On the other hand, in the case of the Anderson localization, it was argued that in the limit of vanishing coupling to the bath a disordered system of weakly interacting one- or two-dimensional electrons cannot support either activated or variable-range hopping transport at low T, undergoing instead a metal-insulator transition at a *finite* critical temperature T_c [7].

With this background in mind, we now formulate the main question we are concerned with here: *What is the relation between pinning and localization?* Which of them restricts the mobility of strongly correlated CDWs more effectively? Specifically, we aim to understand what are the ratios of the characteristic scales in energy space and in real space, $T_c/\Delta_{\rm cl}$ and $\xi_{\rm loc}/\xi_{\rm pin}$, respectively, where $\Delta_{\rm cl}$ is the pinning temperature [8] [defined below in Eq. (4)]. In this Letter, we focus on transport of CDWs in one dimension (1d) in the limit of strong interaction, i.e., on the case of very rigid CDWs, and consider spinless electrons.

Let us specify the model. The spatial modulation of the charge density $\rho(x)$ is written in terms of a smoothly varying phase $\phi(x)$ of a single-harmonic CDW as $\rho = -\partial_x\phi/\pi + \cos\left[2(k_F x - \phi)\right]/\pi\lambda$. The Hamiltonian $H = H_{\rm el} + H_{\rm kin} + H_{\rm dis}$ is given by a sum of the elastic part $H_{\rm el} = c\int dx\,(\partial_x\phi)^2/2$, the kinetic part $H_{\rm kin} = \pi v_F\int dx\,\Pi^2/2$, where Π is the momentum conjugated to ϕ (throughout the paper $\hbar = 1$), and the part $H_{\rm dis}$ describing backscattering of electrons off a static

random potential:

$$H_{\rm dis} = \frac{1}{2\pi\lambda}\int dx\,\left[V_b(x)e^{-2i\phi(x)}+\text{h.c.}\right]\;. \quad (1)$$

The disorder is taken to be of white-noise type with the correlators $\overline{V_b(x)V_b^*(0)} = w\delta(x)$ and $\overline{V_b(x)V_b(0)} = 0$. The elastic constant c and the Fermi velocity v_F are input parameters of the low-energy theory and include Fermi-liquid-type renormalizations coming from the ultraviolet scales. The electron current j is related to Π via the Fermi velocity: $j = v_F\Pi$, so that the velocity of elastic waves is given by $u = (\pi v_F c)^{1/2}$. The system can thus be characterized by two velocities, c (up to $\hbar$, the elastic constant in 1d is velocity) and u, the former describing the static properties of a CDW and the latter its dynamics. Their ratio

$$K = u/\pi c \ll 1 \quad (2)$$

is known as the Luttinger constant, and is the main parameter of our consideration.

The classical limit corresponds to $K \to 0$ with the rigidity $c = \text{const}$. To see the significance of this limit, consider the correlator of the phase ϕ in the Matsubara representation $\mathcal{D}(x,\tau) = \langle\phi(0,0)[\phi(0,0)-\phi(x,\tau)]\rangle$, which in the absence of disorder is given by

$$\mathcal{D}_0(x,\tau) = \frac{K}{4}\ln\left[\left(\frac{u}{\pi T\lambda}\right)^2\sinh\frac{\pi T x_+}{u}\sinh\frac{\pi T x_-}{u}\right], \quad (3)$$

where $x_\pm = x\pm iu\tau$ and a proper ultraviolet cutoff is assumed, e.g., $x\to x+\lambda\,\text{sgn}\,x$. With decreasing K the system rapidly acquires rigidity: at zero T, the long-range order is only broken on large length scales at $\ln(|x|/\lambda)\sim 1/K$. In the classical limit, quantum fluctuations of ϕ vanish and only thermal fluctuations with $\mathcal{D}_0(x,\tau) = T|x|/2c$ are present. At the same time, the classical limit is also the static limit ($H_{\rm kin}\to 0$). In the presence of disorder, the long-range order is broken in the classical limit already at $T = 0$, so that $\mathcal{D}(x,\tau)$, averaged over disorder, becomes a function of $x/\xi_{\rm pin}$. The pinning length $\xi_{\rm pin}$ and the amplitude $\Delta_{\rm cl}$ of elastic-energy fluctuations on the scale of $\xi_{\rm pin}$ (assuming that $\xi_{\rm pin}\gg\lambda$) are

$$\xi_{\rm pin} = \lambda\left(c^2/w\lambda\right)^{1/3}\;,\quad \Delta_{\rm cl} = c/\xi_{\rm pin}\;. \quad (4)$$

Slightly beyond the classical limit, at $0 < K \ll 1$, quantum fluctuations soften the pinning potential by changing the exponent of the pinning length $\xi_{\rm pin}\to\lambda(c^2/w\lambda)^{1/(3-2K)}$. This effect is of little importance in our discussion [and of no importance whatsoever if $K\ll 1/\ln(c^2/w\lambda)$]. The essential quantum effects are the onset of dynamics in the system, with a characteristic velocity u, and the emergence of the *quantum localization.* The latter is characterized by the localization length $\xi_{\rm loc}$ and the "localization temperature" T_1, below which the localization effects become strong [9]. We will see below that the behavior of $\xi_{\rm loc}$ and T_1 as a function of K for $K\ll 1$ is highly nontrivial.

In this Letter, we rely on the conventional bosonization (poorly suited to study the localization and dephasing but conveniently treating strong and weak interaction in the disordered system with essentially the same effort). We explore the large-ω expansion of the ac conductivity to extract the relevant parameters of the system [10].

The conductivity is given by the Kubo formula [11] with the current $j = i\partial_\tau\phi/\pi$:

$$\sigma(\omega,T) = -\frac{1}{i\omega}\frac{e^2v_F}{\pi} - \frac{1}{i\omega}\frac{e^2}{\pi^2}\left\{\int_0^{1/T}d\tau\,e^{i\Omega_n\tau}\times\int dx\,\overline{\left\langle T_\tau\dot\phi(x,\tau)\dot\phi(0,0)\right\rangle}\right\}_{i\Omega_n\to\omega+i0}\;. \quad (5)$$

The averaging over ϕ in Eq. (5) is performed with the replicated action $S = S_0 - S_{\rm dis}$, where

$$S_0 = \frac{c}{2}\sum_m\int dx\int d\tau\left[(\partial_x\phi_m)^2+\frac{1}{u^2}(\partial_\tau\phi_m)^2\right]\;, \quad (6)$$

$$S_{\rm dis} = \frac{w}{(2\pi\lambda)^2}\sum_{m,m'}\int dx\int d\tau\int d\tau'\times\cos\left[2\phi_m(x,\tau)-2\phi_{m'}(x,\tau')\right]\;. \quad (7)$$

Inspection of Eqs. (5)-(7) shows that $\sigma(\omega,T)$ is expandable in powers of disorder as

$$\sigma(\omega,T) = -\frac{e^2v_F}{\pi}\frac{1}{i(\omega+i0)}\sum_{N=0}\left(\frac{\Delta_{\rm cl}}{T}\right)^{3N}\left(\frac{\pi T\lambda}{u}\right)^{2KN}\times f_N(\Omega_n/T)\,|_{i\Omega_n\to\omega+i0}\;, \quad (8)$$

which is at the same time the large-ω expansion. The functions f_N are dimensionless (parametrized by the constant K) real functions of Ω_n/T, with $f_0 = 1$.

At first order in w we have a contribution to $\sigma(\omega,T)$ in terms of $\mathcal{D}_0(x,\tau)$ [Eq. (3)]:

$$\sigma_1(\omega,T) = -\frac{4}{i\omega}\frac{e^2}{\pi^2}\frac{w}{(2\pi\lambda)^2}\times\left[D_n^2\left(2C_0-C_n-C_{-n}\right)\right]_{i\Omega_n\to\omega+i0}\;, \quad (9)$$

where

$$D_{n\neq 0} = \int d\tau\,e^{i\Omega_n\tau}\int dx\,\partial_\tau\mathcal{D}_0(x,\tau) = \frac{i\pi v_F}{\Omega_n}\;, \quad (10)$$

$$C_n = \int d\tau\,e^{i\Omega_n\tau-4\mathcal{D}_0(0,\tau)}\;, \quad (11)$$

$D_0 = 0$. When calculating the difference $2C_0 - C_n - C_{-n}$ in Eq. (9), the ultraviolet cutoff in Eq. (3) can be neglected for $K < 3/2$ and $\sigma_1(\omega,T)$ is then explicitly obtained as

$$\sigma_1(\omega,T) = \frac{2ie^2v_F\Delta_{\rm cl}^3}{\pi T\omega^3}\left(\frac{2\pi T\lambda}{u}\right)^{2K}K^2\Gamma(1-2K)\times\left[\frac{1}{\Gamma^2(1-K)}-\frac{\sin\pi K}{\pi}\frac{\Gamma(K-i\frac{\omega}{2\pi T})}{\Gamma(1-K-i\frac{\omega}{2\pi T})}\right]\;. \quad (12)$$

An important observation is that in Eq. (12) there appears a characteristic scale $\omega \sim KT$. Specifically, for $K \ll 1$ the real part of $\sigma_1(\omega, T)$ is written as

$$\mathrm{Re}\,\sigma_1(\omega, T) \simeq 4e^2 v_F \omega_{\mathrm{pin}}^3 \eta / \pi^3 \omega^2 [\omega^2 + (2\pi KT)^2] \ , \quad (13)$$

where $\omega_{\mathrm{pin}} = u/\xi_{\mathrm{pin}} = \pi K \Delta_{\mathrm{cl}}$ is the pinning frequency and $\eta(\omega, T) = (\lambda \max\{\omega, T\}/u)^{2K}$ is a weak function which describes the renormalization of disorder by quantum fluctuations [can be neglected for $K \ll 1/\ln(u/\lambda \max\{\omega, T\})$]. The scaling of $\mathrm{Re}\,\sigma_1$ in Eq. (13) with the ratio ω/KT reflects the difference in the energy scales characterizing the behavior of $\sigma(\omega, T)$ at zero T and that of the dc conductivity σ_{dc} as a function of T. The characteristic ω for the zero-T ac conductivity is ω_{pin}, whereas the characteristic T for σ_{dc} is $\Delta_{\mathrm{cl}} \sim \omega_{\mathrm{pin}}/K$.

The ac conductivity can be equivalently rewritten as $\sigma(\omega, T) = e^2 v_F / \pi[-i\omega + M(\omega, T)]$, where the current-relaxation rate ReM depends, in general, on ω and T. For high $T \gg \max\{\Delta_{\mathrm{cl}}, T_1\}$, when both pinning and localization are suppressed, the ω dispersion of $\sigma(\omega, T)$ should obey the Drude formula, i.e., all terms in M of order in w higher than one can be neglected. The first-order expansion (13) allows us to extract the Drude-relaxation rate in the CDW state (here and below we omit the factor η):

$$\mathrm{Re}\, M(\omega, T) = 4\omega_{\mathrm{pin}}^3/\pi^2 \left[\,\omega^2 + (2\pi KT)^2\,\right] \ . \quad (14)$$

The dc conductivity for $T \gg \max\{\Delta_{\mathrm{cl}}, T_1\}$ thus reads

$$\sigma_{\mathrm{dc}}(T) = \pi^3 K^2 (e^2 v_F / \omega_{\mathrm{pin}}^3) T^2 = \pi K (e^2 \lambda^2 / w) T^2 \ . \quad (15)$$

The corresponding backscattering time at $\omega = 0$ is

$$\tau(T) = \omega_{\mathrm{pin}}^{-1} \left(\pi T/\Delta_{\mathrm{cl}}\right)^2 \ . \quad (16)$$

The dependence of τ on K for small K is of central importance to us. First of all, let us note that, although σ_{dc} decreases with increasing strength of interaction, τ diverges in the classical limit $K \to 0$, $c = \mathrm{const}$. This divergency suggests that the quantum localization is destroyed at temperature T_1 which is much smaller than the classical scale Δ_{cl}.

To find T_1, we use the general condition, valid in 1d for arbitrary strength of interaction,

$$\tau(T_1) \sim \tau_\phi(T_1) \ , \quad (17)$$

where $\tau_\phi(T)$ is the phase-breaking time [12]. For $\tau_\phi \ll \tau$, the quantum-interference effects are suppressed on the ballistic scale and so reduce to the weak-localization correction to the conductivity (see Ref. 3 for a derivation of this correction at $1 - K \ll 1$), whereas for $\tau_\phi \gg \tau$ the localization, which develops on the scale of the mean free path, is only slightly affected by the inelastic processes. Within the bosonization framework, one of the regular ways to extract τ_ϕ is to proceed to the third order ($N = 3$) in Eq. (8), which is a minimal order that exhibits the Anderson localization [13]. In this paper, however, we do not follow this intricate path and rely on a heuristic argument leading in the limit of strong interaction to

$$\tau_\phi^{-1}(T_1) \sim T_1 \ . \quad (18)$$

For a weakly interacting Luttinger liquid [3], $\tau_\phi^{-1}(T) \sim (1-K)[T/\tau(T)]^{1/2}$, which gives $\tau_\phi^{-1}(T_1) \sim (1-K)^2 T_1$. It is important that (i) soft inelastic scattering with energy transfers $\omega' \ll \tau_\phi^{-1}(T)$ is not effective in dephasing the localization effects (similarly to Fermi liquids [14]) and (ii) characteristic $\omega' \lesssim T$. Hence, the upper bound for $\tau_\phi^{-1}(T)$ is the temperature itself. Piecing together these results [and assuming that $\tau_\phi^{-1}(T)$ is a monotonic function of the interaction strength], we arrive at Eq. (18). Combining Eqs. (16)-(18) we get

$$T_1 \sim K^{1/3} \Delta_{\mathrm{cl}} \ , \quad (19)$$

i.e., the quantum localization sets in when σ_{dc} is already strongly suppressed by the pinning.

To find ξ_{loc}, we first note that the mean free path $l(T)$ at $T \ll \Delta_{\mathrm{cl}}$ turns out to be much smaller than ξ_{pin}. As follows from Eq. (16), for excitations characterized by velocity u: $l(T) = u\tau(T) = \xi_{\mathrm{pin}}(\pi T/\Delta_{\mathrm{cl}})^2$. Since in 1d the localization is established on the scale of the mean free path, we have for the localization length $\xi_{\mathrm{loc}}(T)$ at temperature T_1:

$$\xi_{\mathrm{loc}} \sim K^{2/3} \xi_{\mathrm{pin}} \ . \quad (20)$$

This is the same spatial scale at which the Giamarchi-Schulz [11, 15] renormalization-group flow enters the strong-coupling regime [16]. Note that it is the localization that develops at this scale, not pinning; which is worth emphasizing in view of the Anderson localization length and the pinning length being conventionally thought to be the same in CDWs.

We thus see that both T_1/Δ_{cl} and $\xi_{\mathrm{loc}}/\xi_{\mathrm{pin}}$ are small in CDWs, which is quite remarkable since the relative strength of pinning and localization depends on whether we consider scaling in temperature or in real space. In the ultraviolet limit of large T or small spatial scale L neither pinning nor localization is important. With decreasing T, the classical pinning "wins", i.e., happens at $T \sim \Delta_{\mathrm{cl}} \gg T_1$. A subtle point, however, is that with increasing L, it is the quantum localization that wins, i.e., it develops at $L \sim \xi_{\mathrm{loc}} \ll \xi_{\mathrm{pin}}$. This unusual behavior reflects the peculiarity of the very rigid electron system: the spatial scale ξ_{pin} of the low-energy excitations is much larger than the mean free path $l(T)$ which characterizes their "center-of-mass" diffusion. Counterintuitively, the Anderson localization length ξ_{loc} vanishes to zero in the classical limit. This implies that the conductance at low

T will be strongly suppressed for L larger than $\xi_{\rm loc}$, which is much smaller than $\xi_{\rm pin}$.

We finish with a brief discussion of the T dependence of $\sigma_{\rm dc}$ for $T \ll \Delta_{\rm cl}$. Since in 1d the averaging over disorder assumes summation of random resistances connected in series, the activation over barriers whose typical height is $\Delta_{\rm cl}$ and statistics is Gaussian is described by $\ln\sigma_{\rm dc} \propto -(\Delta_{\rm cl}/T)^2$, rather than by the Arrhenius law. This falloff continues with decreasing T down to the critical temperature $T_c \sim T_1$ [Eq. (19)], below which $\tau_\phi^{-1}(T)$ and $\sigma_{\rm dc}(T)$ vanish to zero [17], similarly to Ref. 7. Since $\xi_{\rm pin}/\xi_{\rm loc}(T) \sim |\ln\sigma_{\rm dc}(T)|$ for $\Delta_{\rm cl} \gg T \gg T_1$, the presence of the quasiclassical barriers does not affect the estimate for T_c.

To summarize, there are two distinctly different disorder-induced phenomena that limit the mobility of CDWs: quantum localization and classical pinning. We have discussed their interplay in the transport properties of CDWs in 1d. The pinning turns out to be stronger in the respect that the pinning temperature $\Delta_{\rm cl}$ is larger than the critical temperature T_c of the localization transition. On the other hand, the localization is stronger in that the quantum localization length $\xi_{\rm loc}$ is shorter than the pinning length $\xi_{\rm pin}$. A rigorous analytical description of the localization of CDWs [in particular, a derivation of Eqs. (17) and (18)] is clearly warranted. More generally, the challenge is to study the dependence of the conductivity $\sigma(\omega, q, T)$ as a function of frequency, momentum, and temperature, where both phenomena and corresponding scales in all three variables will show up. Here, we have addressed the L dependence of the dc conductance and the T dependence of the conductivity at $\omega, q = 0$.

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[1] P.W. Anderson, Phys. Rev. **109**, 1492 (1958).
[2] B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
[3] I.V. Gornyi, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. Lett. **95**, 046404 (2005); Phys. Rev. B **75**, 085421 (2007).
[4] A.I. Larkin, Sov. Phys. JETP **31**, 784 (1970); see also Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975); H. Fukuyama and P.A. Lee, Phys. Rev. B **17**, 535 (1978).
[5] M.V. Feigelman and V.M. Vinokur, in *Charge Density Waves in Solids*, edited by L.P. Gor'kov and G.Grüner (North-Holland, Amsterdam, 1989).
[6] G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994); S. Brazovskii and T. Nattermann, Adv. Phys. **53**, 177 (2004).
[7] I.V. Gornyi, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005); D.M. Basko, I.L. Aleiner, and B.L. Altshuler, Ann. Phys. (N.Y.) **321**, 1126 (2006).
[8] Creep [6], implying that $\Delta_{\rm cl}$ diverges with decreasing driving force, is absent in one dimension.
[9] To recall the results for weak interaction ($a = 1 - K \ll 1$) [3], for spinless electrons in the Luttinger liquid $T_1 \sim (w/a^2u)(u^2/w\lambda)^{2a/(1+2a)}$ and the critical temperature $T_c \sim aT_1/\ln(1/a) \ll T_1$. For $a^2T_1 \lesssim T \lesssim T_1$, $\xi_{\rm loc} \sim (u^2/w)(T\lambda/u)^{2a}$. These results cannot be straightforwardly generalized to the case of strong interaction with $K \ll 1$, in particular because the velocities u and c become then parametrically different from each other.
[10] We focus on the T dependence of the conductivity. The zero-T low-ω conductivity was studied in M.V. Feigelman and V.M. Vinokur, Phys. Lett. **87**A, 53 (1981); I.L. Aleiner and I.M. Ruzin, Phys. Rev. Lett. **72**, 1056 (1994); M.M. Fogler, *ibid.* **88**, 186402 (2002); V. Gurarie and J.T. Chalker, *ibid.* **89**, 136801 (2002), and references therein.
[11] T. Giamarchi, *Quantum physics in one dimension* (Clarendon, Oxford, 2004).
[12] Although Eq. (15) is valid at $T \gg \Delta_{\rm cl}$, using Eq. (16) for the Drude backscattering time is legitimate down to $T \sim T_1$, i.e., as long as the spatial scale u/T that determines $\tau(T)$ (the correlation radius of Friedel oscillations induced by disorder) is shorter than $u\tau(T)$.
[13] The series (8) reproduces (A.D. Mirlin, D.G. Polyakov, and V.M. Vinokur, unpublished) the large-ω expansion for $K = 1$, obtainable, e.g., by the Berezinskii method [V.L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974)], where the localization manifests itself at order $\mathcal{O}(w^3)$.
[14] B.L. Altshuler, A.G. Aronov, and D.E. Khmelnitskii, J. Phys. C **15**, 7367 (1982).
[15] T. Giamarchi and H.J. Schulz, Phys. Rev. B **37**, 325 (1988).
[16] The emergence of a characteristic length scale smaller than $\xi_{\rm pin}$ can also be inferred from the length dependence of the conductance in D.L. Maslov, Phys. Rev. B **52**, R14368 (1995).
[17] Due to a weak electron-phonon coupling, variable-range hopping at $T < T_c$ becomes possible, characterized by the tunneling length $\xi_{\rm tun} \sim K\xi_{\rm pin}$ [18] (for tunneling through the quasiclassical barriers of height $\Delta_{\rm cl}$). Note the sequence of three scales: $\xi_{\rm tun} \ll \xi_{\rm loc} \ll \xi_{\rm pin}$.
[18] A.I. Larkin and P.A. Lee, Phys. Rev. B **17**, 1596 (1978); S.V. Malinin, T. Nattermann, and B. Rosenow, *ibid.* **70**, 235120 (2004).
[19] B. Rosenow, A. Glatz, and T. Nattermann, arXiv:cond-mat/0610426 [Phys. Rev. B (to be published)].